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MASS TRANSFER UNDER CONDITIONS OF FILTRATION IN A RANDOMLY INHOMOGENEOUS MEDIUM

Yu. A. Buevich

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Dispersion phenomena occurring under conditions of filtration in a medium whose porosity and permeability are homogeneous and isotropic random fields are studied. The effective coefficients of hydraulic resistance and the dispersion of the impurity are calculated, and the averaged equations of filtration and convective diffusion are derived.

Dispersion effects in filtration flows are attributed, as a rule, to convective dispersion, owing to the interaction and mixing of elementary streams, appearing in a flow in a criss-crossed pore space, and to hindered molecular diffusion of the impurity in pores [1, 2]. These phenomena are in equal measure characteristic also of macroscopically homogeneous materials, whose porosity and permeability are independent of the coordinates, and of inhomogeneous materials, when the indicated characteristics form random or determinate fields.

The nonuniformity of the properties of a porous medium lead to the appearance of a unique spatially fluctuating motion of the liquid, superposed on the average filtrational flow and called in [3] filtrational pseudoturbulence. The correlation properties of the corresponding random velocity field are studied in [3, 4], and in application to a flow in a closely packed granular bed in [5].

It is obvious that pseudoturbulent motion gives rise to the appearance of an additional convective dispersion of the impurity. Since the pseduoturbulent mixing length is of the order of the linear scale of the inhomogeneities and the latter is usually much larger than the internal structural scale (pore size), pseudoturbulent dispersion of the impurity in real inhomogeneous media often more important than dispersion caused by other mechanisms, even in cases when the amplitude of the fluctuations of the rate of filtration is relatively small, i.e., the inhomogeneity is weak.

Attempts have been made to study the indicated dispersion and to obtain an averaged equation of convective diffusion based on the assumptions that the flow of moles of liquid is a Markovian process and that A. N. Kolmogorov's equations, relating the impurity concentration to the moments of the random field of the fluctuations in the rate of filtration, are valid

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(see [6, 7] and the references quoted there). As a result, a quite complicated integrodifferential equation for the average impurity concentration, whose study and simplification require the use of special localization and regularization techniques, was derived. In addition, in [6, 7] specific Lagrangian correlation functions, whose relationship with the characteristics of the random permeability and porosity fields actually remains unknown, are introduced. In what follows the averaged equations of filtration and convective diffusion in a filtration flow are derived, like in [4, 5], by a considerably simpler and more effective method, based on the correlation theory of stationary stochastic processes [8] and not requiring significant additional assumptions except for the completely natural assumption that the linear scale of the random fields is much smaller than the characteristic scale of the average concentration field.

Stochastic Equations. We shall assume that the local porosity M and permeability K of the porous medium can be represented in the form

$$M = m + m',\ K = k + k',\ \langle m' \rangle = \langle k' \rangle = 0, \tag{1}$$

where m and k are constants, equal to the average porosity and permeability of the medium, respecitvely; and, m' and k' are the uniform and isotropic random fields, whose linear scales ℓ are much larger than the pore size. In this case, the introduction of local porosity and permeability as effective macroscopic characteristics of the medium is correct. On scales greatly exceeding ℓ, the porosity of the medium can thus be regarded as uniform on the average.

In what follows we shall employ the methods of the correlation theory of stationary stochastic processes [8], according to which an arbitrary uniform random field can be represented in the form of the stochastic Fourier–Stieljtes integral

$$\alpha'(\mathbf{r}) = \int \exp(i\varkappa\mathbf{r})\, dZ_\alpha,\quad dZ_\alpha = dZ_\alpha(\varkappa),$$

the correlation function characterizing the uniform random fields α' and β' is expressed in the form

$$R_{\alpha,\beta}(\mathbf{x}) = \langle \alpha'(\mathbf{r})\beta'(\mathbf{r}+\mathbf{x}) \rangle =$$
$$= \iint \exp(-i\varkappa\mathbf{r} + i\varkappa'(\mathbf{r}+\mathbf{x}))\langle dZ^*_\alpha(\varkappa)\, dZ_\beta(\varkappa') \rangle,$$

while the average quantities in the integrands can be formally written as

$$\langle dZ^*_\alpha(\varkappa)\, dZ_\beta(\varkappa') \rangle = \delta(\varkappa - \varkappa')\Phi_{\alpha,\beta}(\varkappa)\, d\varkappa d\varkappa',$$

where $\delta(x)$ is the vector Dirac δ function, and $\Phi_{\alpha,\beta}$ is the joint spectral density of the random fields α' and β'. Thus

$$R_{\alpha,\beta}(\mathbf{x}) = \int \exp(i\varkappa\mathbf{x})\Phi_{\alpha,\beta}(\varkappa)\, d\varkappa,$$

where the integration extends over the volume of the wave space $\varkappa$. The spectral densities $\Phi_{m,m}$, $\Phi_{k,k}$, and $\Phi_{m,k}$ of the random fields m' and k' characterize the nonuniformity of the porous medium, and they must be regarded as known. For simplicity we assumed that these random fields are not only uniform, but also isotropic. Then the indicated spectral densities depend only on the modulus $\varkappa$ of the wave vector. We have

$$\begin{gathered} m_0^2 = R_{m,m}(0) = 4\pi \int_0^\infty \Phi_{m,m}(\varkappa)\varkappa^2 d\varkappa, \\ k_0^2 = R_{k,k}(0) = 4\pi \int_0^\infty \Phi_{k,k}(\varkappa)\varkappa^2 d\varkappa, \\ m_0 k_0 R = R_{m,k}(0) = 4\pi \int_0^\infty \Phi_{m,k}(\varkappa)\varkappa^2 d\varkappa, \end{gathered} \tag{2}$$

where R is the correlation coefficient, while m_0 and k_0 are the rms fluctuations of the porosity and permeability. If M and K are related by a determinate functional relation, then (M = f(K))

$$\begin{gathered} M \approx f + gk' + hk'^2,\ \{f,\ g,\ h\} = \{1,\ d/dK,\ d^2/2dK^2\} f|_{K=k}, \\ m \approx f + h\langle k'^2 \rangle,\ m' \approx gk',\ m_0 = gk_0,\ R = 1, \end{gathered} \tag{3}$$

i.e., in particular, the average porosity m of an inhomogeneous medium with an average permeability k differs from the porosity of a uniform medium with the same permeability, equal to f.

In order to concentrate on the fundamental aspects of the problem and to simplify as much as possible the calculations, we shall write the local equations for filtration and conservation of the impurity mass in the simple form

$$\mathbf{V} = -\frac{K}{\mu}\nabla P,\ \nabla\mathbf{V} = 0,\ M\frac{\partial C}{\partial t} + \nabla(C\mathbf{V}) = 0, \tag{4}$$

which corresponds to the assumption that Darcy's law holds (i.e., filtration is linear) and which neglects the effect of fine-scale dispersion of the impurity, owing to molecular diffusion and mixing in the criss-crossed pore space.

We represent the local velocity $\mathbf{V}$, the local pressure P, and the local corresponding average values $\mathbf{v}$, p, and c and fluctuations $\mathbf{v}'$, p', and c' with zero means. The vectors $\mathbf{v}$ and ∇p can be assumed to be independent of the coordinates and time; then $\mathbf{v}'$ and p' are homogeneous random fields. The average concentration of the impurity c depends, generally speaking, on t and $\mathbf{r}$; therefore, the random field c' is nonuniform and, in addition, depends in a determinate manner on the time.

Substituting the indicated expressions for the local quantities into (4) and averaging, we obtain the equations

$$\begin{gathered}\mathbf{v} = -\frac{k}{\mu}\nabla p - \frac{1}{\mu}\langle k'\nabla p'\rangle,\ \nabla\mathbf{v} = 0,\\ m\frac{\partial c}{\partial t} + \mathbf{v}\nabla c + \left\langle m'\frac{\partial c'}{\partial t}\right\rangle + \nabla\langle c'\mathbf{v}'\rangle = 0.\end{gathered} \tag{5}$$

Subtracting the averaged equations (5) from the corresponding equations in (4) and neglecting quantities of the type $\alpha'\beta' - \langle\alpha'\beta'\rangle$, which we assume are small, we obtain the stochastic equations

$$\mathbf{v}' = -\frac{k}{\mu}\nabla p' - \frac{k'}{\mu}\nabla p,\ \nabla\mathbf{v}' = 0, \tag{6}$$

$$m'\frac{\partial c}{\partial t} + m\frac{\partial c'}{\partial t} + \mathbf{v}'\nabla c + \mathbf{v}\nabla c' = 0, \tag{7}$$

determining the properties of the random fields $\mathbf{v}'$, p', and c'. The quantities $\mathbf{v}$, p, and c in (6) and (7) must be regarded as known.

Filtration. We represent the quantities $\mathbf{v}'$ and p' as Fourier–Stieljtes integrals with the random measures $d\mathbf{Z}_v$, dZ_p. Then Eqs. (6) imply a system of linear equations for the random measures

$$d\mathbf{Z}_v = -i\boldsymbol{\varkappa}\frac{k}{\mu}dZ_p - \frac{1}{\mu}\nabla p\,dZ_k,\ i\boldsymbol{\varkappa}d\mathbf{Z}_v = 0,$$

which have the following solutions:

$$d\mathbf{Z}_v = -\frac{1}{\mu}\left[\nabla p - \frac{(\boldsymbol{\varkappa}\nabla p)\boldsymbol{\varkappa}}{\varkappa^2}\right]dZ_k,\quad dZ_p = \frac{i}{k}\frac{\boldsymbol{\varkappa}\nabla p}{\varkappa^2}dZ_k. \tag{8}$$

From here, taking into account the definition of k_0 in (2),

$$\langle k'\nabla p'\rangle = R_{k,\nabla p}(0) = -\frac{1}{k}\int\frac{(\boldsymbol{\varkappa}\nabla p)\boldsymbol{\varkappa}}{\varkappa^2}\Phi_{k,k}(\varkappa)\,d\boldsymbol{\varkappa} = -\frac{4\pi}{3k}\left(\int_0^\infty\Phi_{k,k}(\varkappa)\,\varkappa^2d\varkappa\right)\nabla p = -\frac{k_0^2}{3k}\nabla p, \tag{9}$$

and after substituting (9) into (5) we obtain an equation describing filtration in a nonuniform porous medium:

$$\mathbf{v} = -\frac{k}{\mu}\left(1 - \frac{k_0^2}{3k^2}\right)\nabla p,\ \nabla\mathbf{v} = 0. \tag{10}$$

The effective permeability of the inhomogeneous medium, determining the coefficient of proportionality between the average rate of filtration and the gradient of the mean pressure, thus turns out to be somewhat lower than the average permeability of the medium. The decrease in the flow of liquid through the region with the reduced permeability is not completely compensated by an increase in the flow through the region whose local permeability exceeds the average value. This can be interpreted as an increase in the hydraulic resistance to the filtrational flow, owing to the nonuniformity of the porous medium. We emphasize that the system studied differs from a pseudofluidized bed – a nonuniform medium with a mobile porous framework, in which the fluctuations of the porosity, caused by the pseduoturbulent motion of the particles, reduces the hydrulic resistance [9].

Using the general method, it is not difficult to obtain a representation for the correlation functions of the random fields $\mathbf{v}'$, p', m', and k' and thereby give an exhaustive statistical description of the properties of filtrational pseudoturbulence on the basis of the correlation theory. In particular, the mean squares of the components of the fluctuation rate of filtration are given by

$$\langle v_1'^2 \rangle = \frac{8}{15}\left(\frac{\nabla p}{\mu}\right)^2 k_0^2, \quad \langle v_2'^2 \rangle = \langle v_3'^2 \rangle = \frac{1}{15}\left(\frac{\nabla p}{\mu}\right)^2 k_0^2, \tag{11}$$

and in addition the r_1 axis is oriented along the vector $-\nabla p \sim \mathbf{v}$. Thus, the longitudinal rms fluctuational rate of filtration is $2\sqrt{2} \approx 2.83$ times higher than the analogous transverse value, which, for example, is in agreement with the result of [5].

Concentration Fluctuations. Since the average concentration c depends (in a determinate manner) on the coordinates and time, the random field c', as is evident from Eq. (7), is inhomogeneous and also depends on the time. To eliminate the time as an independent variable it is sufficient to Laplace transform (7) (s is the transform variable); the initial value c may be set equal to zero without loss of generality. In order to employ in the analysis of the inhomogeneous random field φ' (the transform of the field c') the methods developed for uniform random fields, we introduced the Taylor expansion of the quantities φ and $\nabla\varphi$ appearing in the transformed equation (7), around an arbitrary point selected as the origin of the coordinates, i.e.,

$$\varphi(\mathbf{r}) = \varphi^\circ + \mathbf{r}\nabla\varphi^\circ + \ldots, \quad \nabla\varphi(\mathbf{r}) = \nabla\varphi^\circ + (\mathbf{r}\nabla)\nabla\varphi^\circ + \ldots, \tag{12}$$

where the degree sign indicates that the corresponding quantities are determined at this point; they must be viewed as some constants.

Therefore, the Laplace-transformed Eq. (7) can be written as

$$(sm + \mathbf{v}\nabla)\varphi' + s\varphi m' + \mathbf{v}'\nabla\varphi = (sm + \mathbf{v}\nabla)\varphi' + sm'(1 + \mathbf{r}\nabla + \ldots)\varphi^\circ + \mathbf{v}'(1 + \mathbf{r}\nabla + \ldots)\nabla\varphi^\circ = 0. \tag{13}$$

If $\alpha'(\mathbf{r})$ is a uniform random field, represented in the form of a Fourier–Stieltjes integral with the random measure dZ_α, then $\mathbf{r}\alpha'(\mathbf{r})$ can also be expressed in this form, and in addition the random measure will be given by $id(dZ_\alpha/d\varkappa)$. The random fields represented by the product of α' by the components of $\mathbf{r}$ in powers higher than unity can be represented analogously. For this reason, from (13) we can obtain by the previous methods a relation for the random measure of the field φ':

$$dZ_\varphi = -\frac{1}{sm + i\varkappa\mathbf{v}}\left[s\varphi dZ_m + is\frac{\partial\varphi}{\partial r_j} d\left(\frac{\partial Z_m}{\partial\varkappa_j}\right) + \frac{\partial\varphi}{\partial r_j} dZ_{v,j} + i\frac{\partial^2\varphi}{\partial r_j \partial r_k} d\left(\frac{\partial Z_{v,j}}{\partial\varkappa_k}\right) + \ldots\right], \tag{14}$$

where the repeated indices are summed. The degree sign is dropped in (14), and in addition $\mathbf{r}$ is once again understood to be the radius vector of an arbitrary point. The relations (8) and (14) enable determining, in principle, the mean values $\langle dZ^*_m dZ_\varphi \rangle$ and $\langle dZ^*_v dZ_\varphi \rangle$, and then calculating the quantities $s\langle m'\varphi' \rangle$ and $\langle \varphi'\mathbf{v}' \rangle$.

We assume below that the ratio of the linear scales of the nonuniformity and the field of the average concentration $\ell/L \ll 1$; as follows from the second equation in (5), the time scale of this field equals L/v. The first two terms in the indicated equation evidently are of the order of $cv\ell/L$, and the next two terms are of the order of $cv\ell/L^2$. Neglecting terms in the expressions for the quantities $\langle m'\partial c/\partial t \rangle$ and $\nabla\langle c'\mathbf{v}' \rangle$, which are of higher order in ℓ/L, is equivalent to retaining in the indicated equation for the average concentration only the terms which contain derivatives of s with respect to t and $\mathbf{r}$ no higher than of second order. We shall confine ourselves to this accuracy, which is entirely natural in the analysis of processes of the diffusion type. It is clear that this imposes restrictions on the accuracy of the calculation of the average quantities in the second equation of (5) or their transforms in (13), and therefore also on the number of terms in the expansions (12) which must be taken into account in (13) and then in (14). It turns out that all terms in these expansions except for the first terms lead to the appearance of quantities which are proportional to derivatives of third and higher orders of the average concentration in the final equations of convective diffusion, and they must be dropped in accordance with the accuracy adopted for the description of the diffusion process.

In integrating over the volume of the wave space it is conventient to introduce a spherical coordinate system with the polar axis $\varkappa_1$, oriented along the vector $-\nabla p \sim v$, and the axis $\varkappa_2$ chosen so that the vector ∇c would lie in the plane $(\varkappa_1, \varkappa_2)$. Then the volume element in wave space is given by $d\varkappa = \varkappa^2 \sin\theta d\varkappa d\theta d\eta$. Making the substitution $\psi = \cos\theta$ we obtain

$$s\langle m'\varphi'\rangle = \frac{s^2}{m}\int_0^{2\pi} d\eta \int_{-1}^{1} d\psi \int_0^{\infty} \frac{\varkappa^2 d\varkappa}{s^2 + (\varkappa v/m)^2\psi^2} \times [-s\varphi\Phi_{m,m}(\varkappa) + (1-\psi^2)\nabla\varphi\cdot(\nabla p/\mu)\Phi_{m,k}(\varkappa)].$$

The integrals over $d\psi$ are functions of $sm/\varkappa v$, which since large values of the diffusion time correspond to small values of s, can be expanded in a series in powers of this quantity. In so doing, only the terms in these series, which after transforming back to the original functions yield quantities containing derivatives of c with respect to t and r of order no higher than second, need be retained. After the calculations are completed and after transforming to the original functions we obtain

$$\left\langle m'\frac{\partial c'}{\partial t}\right\rangle = -\frac{\pi}{2}\left(m_0^2\frac{l_m}{v}\frac{\partial^2 c}{\partial t^2} + \gamma R\frac{m_0 k_0}{k} l_{mk}\frac{\partial^2 c}{\partial t\partial x}\right), \tag{15}$$

where the axis $r_1 = x$ is oriented along the vector $-\nabla p \sim v$. Retaining only the terms with the first derivatives of c we obtain analogously for the longitudinal x-component of the pseudoturbulent flow $q = \langle c'v'\rangle$:

$$q_x = \langle c'v_x'\rangle = -\frac{\pi}{2}\left(\gamma R\frac{m_0 k_0}{k} l_{mk}\frac{\partial c}{\partial t} + \gamma^2\frac{k_0^2}{k^2} v l_k\frac{\partial c}{\partial x}\right). \tag{16}$$

The transverse y component of the flow q is proportional to the mixed derivative $d^2c/dtdy$, and on the basis of the accuracy adopted here it must be set equal to zero. In deriving (15) and (16) we employed the definitions (2) and the equations (8), and we also introduced the notation

$$\begin{gathered} l_m = \frac{4\pi}{m_0^2}\int_0^{\infty}\Phi_{m,m}(\varkappa)\varkappa d\varkappa, \quad l_{mk} = \frac{4\pi}{Rm_0 k_0}\int_0^{\infty}\Phi_{m,k}(\varkappa)\varkappa d\varkappa, \\ l_k = \frac{4\pi}{k_0^2}\int_0^{\infty}\Phi_{k,k}(\varkappa)\varkappa d\varkappa, \quad \gamma = \left(1 - \frac{k_0^2}{3k^2}\right)^{-1}. \end{gathered} \tag{17}$$

In the general case $\ell_m \sim \ell_{mk} \sim \ell_k \sim \ell$ and in the presence of a determinate functional dependence between the local porosity and permeability all these scales are the same, $R = 1$, and m_0 can be expressed in terms of k_0 in accordance with (3).

Using standard techniques, it is not difficult to calculate also the two-point correlation functions, relating the random field c' with the other random fields introduced above.

Convective Diffusion. Using (15) and (16), we obtain from (5) the following equation describing on the average the convective diffusion of the impurity in the nonuniform porous medium:

$$\begin{gathered} m\frac{\partial c}{\partial t} + v\frac{\partial c}{\partial x} = T\frac{\partial^2 c}{\partial t^2} + a\frac{\partial^2 c}{\partial t\partial x} + D\frac{\partial^2 c}{\partial x^2}, \\ T = \frac{\pi}{2}m_0^2\frac{l_m}{v}, \quad a = \pi\gamma R\frac{m_0 k_0}{k} l_{mk}, \quad D = \frac{\pi\gamma^2}{2}\frac{k_0^2}{k^2} v l_k. \end{gathered} \tag{18}$$

Equation (18) has a somewhat nonstandard form, because it contains a mixed derivative of c. To eliminate this derivative we used the method of operator expansions, employed previously in [10, 11] in the derivation of the "equivalent" equation describing heat transfer in a filtrational flow and filtration in fractured porous media. We represent (18) in the form

$$v\frac{\partial c}{\partial x}=\left(1-\frac{a}{v}\frac{\partial}{\partial t}\right)^{-1}\left(-m\frac{\partial c}{\partial t}+T\frac{\partial^2 c}{\partial t^2}+D\frac{\partial^2 c}{\partial x^2}\right)=$$
$$=\left(1+\frac{a}{v}\frac{\partial}{\partial t}+\dots\right)\left(-m\frac{\partial c}{\partial t}+T\frac{\partial^2 c}{\partial t^2}+D\frac{\partial^2 c}{\partial x^2}\right).$$

For the accuracy adopted above, it is easy to derive from here the equation

$$\tau\frac{\partial^2 c}{\partial t^2}+m\frac{\partial c}{\partial t}+v\frac{\partial c}{\partial x}=D\frac{\partial^2 c}{\partial x^2},$$
$$\tau=\frac{ma}{v}-T=\frac{\pi}{v}\left(\gamma Rm\frac{m_0k_0}{k}l_{mk}-\frac{1}{2}m_0^2l_m\right). \tag{19}$$

For $\tau > 0$ this equation is of the hyperbolic type, and for $\tau < 0$ it is of the elliptic type. If $M = f(K)$, i.e., the relations (3) holds, then usually $m_0/m < k_0/k$ [1], so that the parameter $\tau > 0$ and can be interpreted as a relaxation time. With the same accuracy, we can write

$$m\frac{\partial c}{\partial t}=\left(1-\frac{a}{m}\frac{\partial}{\partial x}\right)^{-1}\left(-v\frac{\partial c}{\partial x}+T\frac{\partial^2 c}{\partial t^2}+D\frac{\partial^2 c}{\partial x^2}\right)=$$
$$=\left(1+\frac{a}{m}\frac{\partial}{\partial x}+\dots\right)\left(-v\frac{\partial c}{\partial x}+T\frac{\partial^2 c}{\partial t^2}+D\frac{\partial^2 c}{\partial x^2}\right),$$

whence, instead of (19), it follows that

$$m\frac{\partial c}{\partial t}+v\frac{\partial c}{\partial x}=T\frac{\partial^2 c}{\partial t^2}+D_1\frac{\partial^2 c}{\partial x^2},$$
$$D_1=D-\frac{va}{m}=\pi\gamma v\left(\frac{\gamma}{2}\frac{k_0^2}{k^2}l_k-R\frac{m_0k_0}{k}l_{mk}\right). \tag{20}$$

This equation can also be, generally speaking, both hyperbolic and elliptic. If $M = f(K)$, it is more likely that it will be of the elliptic type.

Moreover, setting $a = \sigma_1 + \sigma_2$, in the general case Eq. (18) can be rewritten in the form

$$m\left(1-\frac{\sigma_1}{m}\frac{\partial}{\partial x}\right)\frac{\partial c}{\partial t}+v\left(1-\frac{\sigma_2}{v}\frac{\partial}{\partial t}\right)\frac{\partial c}{\partial x}=T\frac{\partial^2 c}{\partial t^2}+D\frac{\partial^2 c}{\partial x^2},$$

whence by the previous method, within the stated accuracy, we obtain

$$m\frac{\partial c}{\partial t}=-v\left(1-\frac{\sigma_2}{v}\frac{\partial}{\partial t}\right)\frac{\partial c}{\partial x}+T\frac{\partial^2 c}{\partial t^2}+\left(D-\frac{\sigma_1 v}{m}\right)\frac{\partial^2 c}{\partial x^2},$$

and then

$$v\frac{\partial c}{\partial x}=-m\frac{\partial c}{\partial t}+\left(T-\frac{\sigma_2 m}{v}\right)\frac{\partial^2 c}{\partial t^2}+\left(D-\frac{\sigma_1 v}{m}\right)\frac{\partial^2 c}{\partial x^2}.$$

It is easy to see that σ_1 or σ_2 can be chosen so that the coefficient in front of either of the second derivatives vanishes, and as a result we arrive at equations of the parabolic type

$$m\frac{\partial c}{\partial t}+v\frac{\partial c}{\partial x}=D_*\frac{\partial^2 c}{\partial x^2},\quad D_*=D-\frac{v}{m}\left(a-\frac{v}{m}T\right) \tag{21}$$

or

$$m\frac{\partial c}{\partial t}+v\frac{\partial c}{\partial x}=T_*\frac{\partial^2 c}{\partial t^2},\quad T_*=T-\frac{m}{v}\left(a-\frac{m}{v}D\right). \tag{22}$$

Thus, within the limits of the physical accuracy of the model under study, which reduces the description of the dispersion process to the solution of second-order partial differential equations, even the concept of the type of equation is, in a certain sense relative. The differences between the solutions of Eqs. (18)-(22) fall within the limits of error of the theory itself, and the specific form of the equation of convective diffusion with given boundary and initial conditions must be chosen so that the corresponding boundary value problem would be properly posed.

We note in this connection that the equivalent equations for heat transfer in a heterogenerous medium in the presence of filtrational flow [10] and filtration in a fissured-porous medium [11], which are of the elliptic type, can also be transformed to equations of the parabolic type, which is important in that boundary value problems in the theory of heat transfer and filtration be properly posed.

The equations of convective diffusion obtained above do not contain derivatives with respect to the transverse coordinates, since the transverse components of the pseudoturbulent flow of the impurity vanish. More accurately, these components differ from zero (they depend, for example, on the corresponding components of the gradient of the quantity $\partial c/\partial t$), but are higher order infinitesimals and should not be taken into account in the approximation considered. This can be interpreted as the absence of statistical independence between the random fields of the fluctuations of the concentration and the transverse components of the rate of filtration. In reality, however, as is evident from Eq. (7) or the expression (14), the indicated random fields are interrelated. For this reason, it is more correct to state that the fluctuations of the impurity concentration are such that the pseudoturbulent flow of the impurity, owing to the moles of matter moving in the direction of the transverse part of the gradient of the average concentration, is precisely compensated by the flow associated with the moles of matter moving in the opposite direction. The existence of exchange between the indicated moles should cause this balance to break down. Therefore, it may be expected that filtrational pseudoturbulence will lead to the appearance of a pseudoturbulent component of the dispersion is taken into account at the outset in the equation of balance of the impurity in (4).

NOTATION

a, coefficient in (18); c is the concentration; D is the dispersion coefficient (diffusion); f, g, and h, coefficients in the expansion (3); k, permeability; L and ℓ, linear scales of the mean concentration field and nonuniformity of the medium, respectively; m, porosity; p, pressure; $R_{\alpha,\beta}$, R, correlation function and the correlation coeficients; $\mathbf{r}$, radius vector; $\mathbf{q}$, pseudoturbulent flow; s, Laplace transform variable; T, a coefficient in (18); t, time; $\mathbf{v}$, rate of filtration; dZ_α, random measure; γ, a parameter in (17); θ, η, polar and azimuthal angles in wave space; $\boldsymbol{\varkappa}$, wave vector; μ, viscosity; τ, relaxation time; $\Phi_{\alpha,\beta}$, spectral density; $\psi = \cos\theta$; the random fields with zero mean are denoted by a prime, and their rms amplitudes are denoted by a zero subcript; a superscript asterisk denotes complex conjugation.

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